Evaluation of System Reliability with Common-Cause Failures, by a Pseudo-Environments Model

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Key Words — **Common-cause failure, Factoring method, Self-failure, System reliability**

Reader Aids —
Purpose: Widen the state of the art
Special math needed for explanations: Probability
Special math needed to use results: Same
Result useful to: Nuclear power system & reliability analysts

Abstract — **This article presents an efficient method to calculate system reliability with CCFs (Common-Cause Failures) by applying the factoring (total probability) theorem when the system and its associated class of CCFs are both arbitrary. Existing methods apply this theorem recursively until no CCF remains to be considered, and so might be time-consuming in computation. Our method applies such a theorem only once and can be carried out in two steps: 1) determine each state in terms of the occurrence (or not) of every CCF in the associated class, to regard it as a pseudo-environment and to calculate its probability or weight; 2) determine each resulting subsystem of the system under the environment and calculate its reliability as in the "no CCF" case and take the weighted sum of such reliabilities, viz, the system reliability. This method is in terms of a Markov process and requires only the occurrence rate of each CCF to obtain the probability of each environment and only the failure rate of each component to obtain the system reliability under each environment, hence is practical, efficient, and useful.**

1. INTRODUCTION

For ease of computation, system reliability evaluation usually assumes that components are subject to self-failures only. But components are often further subject to common-cause-failures (CCFs). Failure to consider CCFs results in exaggerating system reliability. Hence many methods of system reliability evaluation with CCFs have been developed [1-18]. Most calculate the reliability and/or availability of a "parallel" system of 2, 3, or 4 components in terms of Markov processes [2-7], some calculate the availability of several special systems in terms of more general regenerative processes or the technique of supplementary variables [8-10].

Those few methods which are concerned with an arbitrary system are all in terms of Markov processes [11-18]. Chae & Clark [11] considered *s*-identical components and presented a method using the inclusion & exclusion rule under the assumption that different failures (self- or CCFs) are *s*-independent. Dhillon [10, 12] and Fleming [13] also considered *s*-identical components and presented a method in terms of a parameter which is the fraction of self-failures among total failures. Such methods should be further improved before they can be applied to any general case. Other methods calculate system reliability and/or availability by applying the factoring (total probability) theorem. They are distinguished into the following three classes:

1. Each CCF is confined to a subset of either a minimal cut [15] or a "parallel" subsystems of *s*-identical components [14].
2. Those subsets of components which are subject to CCF are pairwise disjoint, vs not pairwise disjoint [17].
3. Completely arbitrary [16, 18].

Although methods in [16, 18] belong to class 3, they all have to apply the factoring theorem recursively until no CCF remains to be considered, and so might be time-consuming. In order to save computation time or improve the efficiency, any method which applies the factoring theorem should apply it only once. Although methods in [15, 17] apply such a theorem only once, they belong to classes 1-2. Besides, they are cumbersome and also receive some dispute on logic consistence due to the complexity induced by considering repair times. We present a method which belongs to class 3; it applies the factoring theorem only once in the following two steps:

1. Look into the class of CCFs first; regard each state in terms of the occurrence (or not) of every CCF as a pseudo-environment E and calculate its weight $\Pr\{E\}$.
2. Calculate $\Pr\{\text{system normal}|E\}$ which is the system reliability under E (as in the no CCF case) and the system reliability is then the weighted sum $\Sigma_E \Pr\{E\} \cdot \Pr\{\text{system normal}|E\}$. The $\{C_i : 1 \le i \le m\}$ of subsets of components which are subject to CCF must be listable such that $C_1, C_2, \ldots, C_K$, $\cup_{j \ge K} C_j$ are pairwise disjoint and $C_{K+1}, C_{K+2}, \ldots, C_m$ are not pairwise disjoint for a unique $0 \le K \le m$.

2. NOTATION, DEFINITION AND BASIC ASSUMPTIONS

Notation

CCF	common-cause failure
S	system identifier
$S(\bar{i}, \bar{j}, \ldots, \bar{k})$	the subsystem of S after deleting those components $i, j, \ldots, k$
p_i	reliability of component i when i is isolated
$\bar{C}$	CCF of components in the subset C
$\{C_i : 1 \le i \le m\}$	the class of all subsets of components in S which are subject to CCF, $\bar{C}_i$'s
x_α	$\{x_1, x_2, \ldots, x_\alpha\}$

P_S system reliability with CCF
P_S^I system reliability when CCF is not considered at all
λ_i failure intensity of a Poisson process for isolated component i
λ_{c_i} occurrence intensity of a Poisson process for $\bar{C}_i$
Other, standard notation is given in "Information for Readers & Authors" at the rear of each issue.

Basic Assumptions

1. System and its components, when isolated, transit between two states, N (Normal) and F (Failed).
2. Besides self-failures, components are subject to CCF, $\bar{C}_i$, for $1 \le i \le m$.
3. The occurrence of the failure (self-failure) of each isolated component i follows a Poisson process with failure intensity λ_i.
4. The occurrence of each $\bar{C}_i$ follows a Poisson process with occurrence intensity λ_{C_i}. Such Poisson processes are s-independent.
5. Components self-failures are s-independent whenever no CCF exists.

3. THE APPROACH TO CALCULATE SYSTEM RELIABILITY

Our method to calculate system reliability with CCF by applying the factoring theorem can be described in two main steps.

1. Determine each environment from the associated class of CCFs and calculate its probability or weight in 3 sub-steps:

1.1 List $C_1, C_2, \ldots, C_m$ according to:

- $C_1, C_2, \ldots, C_K$ & $\cup_{j>K} C_j$ are pairwise disjoint
- $C_{K+1}, C_{K+2}, \ldots, C_m$ are not pairwise disjoint

for a unique K with $0 \le K \le m$ first, and where $K = 0$ means that $C_1, C_2, \ldots, C_m$ are pairwise disjoint (This can always be done according to lemma A in the appendix). Then $C_1, C_2, \ldots, C_m$ satisfy the following properties;

i. The occurrence (or not) of $\bar{C}_1, \bar{C}_2, \ldots, \bar{C}_K$ are s-independent
ii. The occurrence of $\bar{C}_{K+1}, \bar{C}_{K+2}, \bar{C}_m$ are s-independent in the sense that the occurrence of any one, say $\bar{C}_j$, will prevent $\bar{C}_i$ for $C_i C_j \ne \phi$ from happening. Hence, for each $j > K$, we let $\bar{C}_j$ -first denote the event that $\bar{C}_j$ occurs first among those $\bar{C}_x$ for all $x > K$.
iii. For each $j > K$, the occurrence of $\bar{C}_1, \bar{C}_2, \ldots, \bar{C}_K$, $\bar{C}_{K+1}$-first, ..., $\bar{C}_m$-first are s-independent.

1.2 Determine each environement from $C_1, C_2, \ldots, C_k$, $C_{k+1}, \ldots, C_m$ by a vector $(\boldsymbol{x}_K;y)$ or $(\boldsymbol{x}_K;y;t)$ such that —

$$x_i = \begin{cases} 1, & \text{if } \bar{C} \text{ happens (during } [0,t]) \\ 0, & \text{otherwise} \end{cases}$$

$$y = \begin{cases} j & \text{if } \bar{C}_j\text{-first happens among } C_i\text{'s for } i > k \text{ (during } [0,t]) \\ 0 & \text{if no } \bar{C}_j \text{ happens for all } j > K \text{ (during } [0,t]) \end{cases}$$

If $K = m$, then the environment is denoted by $(x_1, x_2, \ldots, x_m)$ or $(x_1, x_2, \ldots, x_m; t)$.

1.3 Calculate the weight of each $(\boldsymbol{x}_K;y;t)$:

$w(\boldsymbol{x}_K;y;t) \equiv \Pr\{\boldsymbol{x}_K;y;t\}$ by

$$w(\boldsymbol{x}_K;y;t) = [\Pi_{1 \le i \le K}\, w_i(x_i;t)] \cdot w(y;t) \qquad (1.3\text{-}1)$$

$$w_i(x_i;t) \equiv \begin{cases} \exp(-\lambda_{C_i} t), & \text{if } x_i = 0 \\ 1\text{-}\exp(-\lambda_{C_i} t), & \text{if } x_i = 1 \end{cases} \qquad (1.3\text{-}2)$$

is the probability that $\bar{C}$ occurs (does not occur) during $[0,t]$ if $x_i = 1$ (resp. $x_i = 0$).

$$\Lambda_{i,K} \equiv \sum_{i>K} \lambda_{C_i}$$

$$w(y;t) \equiv \begin{cases} \exp(-\Lambda_{i,K} t), & \text{if } y = 0 \\ (\lambda_{C_j}/\Lambda_{i,K}) \cdot (1\text{-}\exp(-\Lambda_{i,K} t), & \text{if } y = j \end{cases} \qquad (1.3\text{-}3)$$

is the probability that no $\bar{C}_i$ occurs for all $i > K$ ($\bar{C}_j$-first occurs among those $\bar{C}_i$ for $i > K$) during $[0,t]$ if $y = 0$ ($y = j$). Eq.(1.3-3) follows from lemma E in the appendix; (1.3-2) from basic assumption 4; and (1.3-1) from iii.

2. Calculate reliability in 3 sub-steps:
2.1 Determine each subsystem $S|(\boldsymbol{x}_K;y)$ of S by conditioning on the environment $(\boldsymbol{x}_K;y)$
2.2 Calculate its reliability $P_{S|(\boldsymbol{x}_K;y)}^I(t)$.
2.3 Calculate —

$$P_S(t) = \Sigma_{(\boldsymbol{x}_K;y)}\, w(\boldsymbol{x}_K;y;t) \cdot P_{S|(\boldsymbol{x}_K;y)}^I(t)$$

4. NUMERICAL EXAMPLES

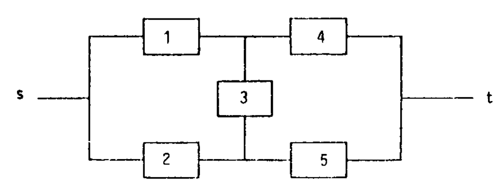

Figure 1. A Bridge Network

4.1 *Example* 1

No CCF happens in figure 1. Hence component self-failures are s-independent, and so —

$$P_S^I = p_3 P_{S(3)}^I + (1-p_3) P_{S(\bar{3})}^I$$

where $S(3)$ denotes the subsystem of S by conditioning on the normal state of 3, $p_i(t) = e^{-\lambda_i t}$ for all $1 \le i \le 5$ and —

$$P_{S(3)}^I = (p_1+p_2-p_1p_2) \cdot (p_4+p_5-p_4p_5)$$

$$P_{S(\bar{3})}^I = p_1p_4+p_2p_5-p_1p_2p_4p_5.$$

Let $\lambda_{ij} = \lambda_{\{i,j\}}$ for convenience.

Example 2
Two CCFs $\overline{12}$ & $\overline{34}$ happen in Figure 1.

Step 1

1.1 $C_1 = \{1,2\}$ & $C_2 = \{3,4\}$ $(m = 2$ & $K = 0)$
1.2 Environments are: (0,0), (1,0), (0,1) & (1,1).
1.3 Weight of each environment:

$$w(0,0;t) = w_1(0;t)w_2(0;t) = \exp(-(\lambda_{12}+\lambda_{34})t)$$

$$w(1,0;t) = w_1(1;t)w_2(0;t) = (1\text{-}\exp(-\lambda_{12}t))\cdot\exp(-\lambda_{34}t)$$

$$w(0,1;t) = w_1(0;t)w_2(1;t) = \exp(-\lambda_{12}t)\cdot(1\text{-}\exp(\lambda_{34}t))$$

$$w(1,1;t) = w_1(1;t)w_2(1;t) = (1\text{-}\exp(-\lambda_{12}t)) \cdot(1\text{-}\exp(-\lambda_{34}t))$$

Step 2

2.1 Resulting subsystems are: $S|(0,0) = S$, $S|(1,0) = S(\bar{1},\bar{2})$, $S|(0,1) = S(\bar{3},\bar{4})$, $S|(1,1) = S(\bar{1},\bar{2},\bar{3},\bar{4})$.
2.2 $P_{S|(0,0)}^I = P_S^I = p_3(p_1+p_2-p_1p_2)(p_4+p_5-p_4p_5) + (1-p_3)(p_1p_4+p_2p_5-p_1p_2p_5)$
$P_{S|(1,0)}^I = P_{S(\bar{1},\bar{2})}^I = 0$, because $\{1,2\}$ is a minimal cutset
$P_{S|(1,1)}^I = P_{S(\bar{1},\bar{2},\bar{3},\bar{4})}^I = 0$
$P_{S|(0,1)}^I = P_{S(\bar{3},\bar{4})}^I = p_2p_5$
2.3 Calculate —

$$P_S = \Sigma\, w(x_1,x_2;t) \cdot P_{S|(x_1,x_2)}^I(t) = \exp(-(\lambda_{12}+\lambda_{34})t) \cdot P_S^I(t) + \exp(-\lambda_{12}t) \cdot (1\text{-}\exp(-\lambda_{34}t)) \cdot p_2p_5$$

The procedure to calculate the system reliability is illustrated via the modified success tree in figure 2.

Example 3

Step 1

1.1 $C_1 = \{4,5\}$, $C_2 = \{1,2\}$, $C_3 = \{1,3\}$ & $C_4 = \{2,3\}$ $(m = 4$ & $K = 1)$
1.2 Environments are:
(0;0), (0;2), (0;3), (0;4), (1;0), (1;2), (1;3) & (1;4).

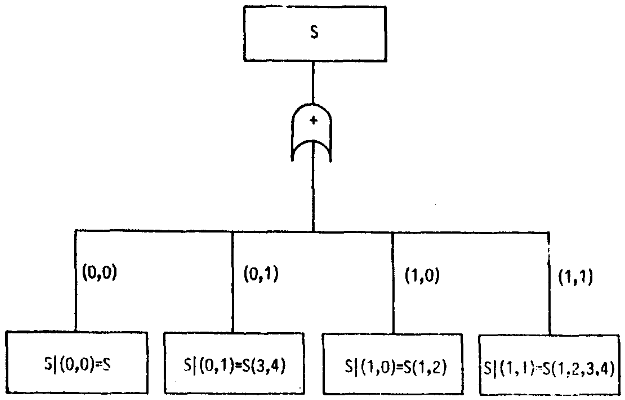

Figure 2. Reliability Evaluation of Example 1 Illustrated in the Modified Success Tree

1.3 $w(0;0;t) = w_1(0;t) \cdot \exp(-(\lambda_{12}+\lambda_{13}+\lambda_{23})t) = \exp(-(\lambda_{45}+\lambda_{12}+\lambda_{13}+\lambda_{23})t)$

$w(0;2;t) = w_1(0;t) \cdot (\lambda_{12}/(\lambda_{12}+\lambda_{13}+\lambda_{23})) \cdot (1-\exp(-(\lambda_{12}+\lambda_{13}+\lambda_{23})t)) = \exp(-\lambda_{45}t) \cdot (\lambda_{12}/(\lambda_{12}+\lambda_{13}+\lambda_{23}))\cdot(1-\exp(-(\lambda_{12}+\lambda_{13}+\lambda_{23})t))$

$w(0;3;t) = w_1(0;t) \cdot (\lambda_{13}/(\lambda_{12}+\lambda_{13}+\lambda_{23})) \cdot (1-\exp(-(\lambda_{12}+\lambda_{13}+\lambda_{23})t)) = \exp(-\lambda_{45}t) \cdot (\lambda_{13}/(\lambda_{12}+\lambda_{13}+\lambda_{23}))\cdot(1-\exp(-(\lambda_{12}+\lambda_{13}+\lambda_{23})t))$

$w(0;4;t) = w_1(0;t) \cdot (\lambda_{23}/(\lambda_{12}+\lambda_{13}+\lambda_{23})) \cdot (1-\exp(-(\lambda_{12}+\lambda_{13}+\lambda_{23})t)) = \exp(-\lambda_{45}t) \cdot (\lambda_{23}/(\lambda_{12}+\lambda_{13}+\lambda_{23}))\cdot(1-\exp(-(\lambda_{12}+\lambda_{13}+\lambda_{23})t))$

$w(1;0;t) = w_1(1;t) \cdot \exp(-(\lambda_{12}+\lambda_{13}+\lambda_{23})t) = (1-\exp(-\lambda_{45}t)) \cdot \exp(-(\lambda_{12}+\lambda_{13}+\lambda_{23})t)$

$w(1;2;t) = w_1(1;t) \cdot (\lambda_{12}/(\lambda_{12}+\lambda_{13}+\lambda_{23})) \cdot (1-\exp(-(\lambda_{12}+\lambda_{13}+\lambda_{23})t)) = (1-\exp(-\lambda_{45}t)) \cdot (\lambda_{12}/(\lambda_{12}+\lambda_{13}+\lambda_{23}))\cdot(1-\exp(-(\lambda_{12}+\lambda_{13}+\lambda_{23})t))$

$w(1;3;t) = w_1(1;t) \cdot (\lambda_{13}/(\lambda_{12}+\lambda_{13}+\lambda_{23})) \cdot (1-\exp(-(\lambda_{12}+\lambda_{13}+\lambda_{23})t)) = (1-\exp(-\lambda_{45}t)) \cdot (\lambda_{13}/(\lambda_{12}+\lambda_{13}+\lambda_{23}))\cdot(1-\exp(-(\lambda_{12}+\lambda_{13}+\lambda_{23})t))$

$w(1;4;t) = w_1(1;t) \cdot (\lambda_{23}/(\lambda_{12}+\lambda_{13}+\lambda_{23})) \cdot (1-\exp(-(\lambda_{12}+\lambda_{13}+\lambda_{23})t)) = (1-\exp(-\lambda_{45}t)) \cdot (\lambda_{23}/(\lambda_{12}+\lambda_{13}+\lambda_{23}))\cdot(1-\exp(-(\lambda_{12}+\lambda_{13}+\lambda_{23})t))$

Step 2

2.1 Resulting subsystems are: $S|(0;0) = S$, $S|(0;2) = S(\bar{1},\bar{2})$, $S|(0;3) = S(\bar{1},\bar{3})$, $S|(1;0) = S(\bar{4},\bar{5})$, $S|(1;2) = S(\bar{1},\bar{2},\bar{4},\bar{5})$, $S|(1;3) = S(\bar{1},\bar{3},\bar{4},\bar{5})$, $S|(1;4) = S(\bar{2},\bar{3},\bar{4},\bar{5})$
2.2 $P_{S|(0;0)}^I = P_S^I$
$P_{S|(0;2)}^I = P_{S(\bar{1},\bar{2})}^I = 0$ due to the fact that $\{1,2\}$ is a minimal cutset

$P^I_{S|(0;3)} = P^I_{S(\bar{1},\bar{3})} = p_2 p_5$

$P^I_{S|(0;4)} = P^I_{S(\bar{2},\bar{3})} = p_1 p_4$

$P^I_{S|(1;0)} = P^I_{S(\bar{4},\bar{5})} = 0$ due to the fact that $\{4,5\}$ is a minimal cutset

$P^I_{S|(1;2)} = P^I_{S(\bar{4},\bar{5},\bar{1},\bar{2})} = 0$

$P^I_{S|(1;3)} = P^I_{S(\bar{4},\bar{5},\bar{1},\bar{3})} = 0$

$P^I_{S|(1;4)} = P^I_{S(\bar{4},\bar{5},\bar{2},\bar{3})} = 0$

2.3 $P_S(t) = \Sigma\, w(x_1;y;t) \cdot P^I_{S|(x_1;y)}(t)$
$= w(0;0;t) \cdot P^I_S(t) + w(0;3;t) \cdot P^I_{S(\bar{1},\bar{3})}(t)$
$+ \Pr\{(0;4;t)\} \cdot P^I_{S(\bar{2},\bar{3})}(t).$

The procedure to calculate the sytem reliability illustrated via the modified success tree in figure 3.

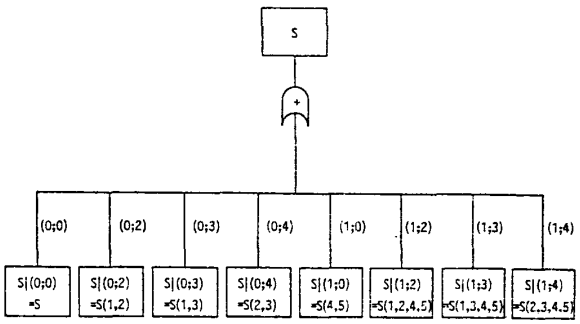

Figure 3. Reliability Evaluation of Example 3 Illustrated in the Modified Success Tree

Suppose that $\lambda_i = 0.05$ for all $1 \le i \le 5$ and $\lambda_C = 0.005$ for all CCF $\bar{C}$'s. Then —

Example		$t = 10$	$t = 100$	$t = 200$
1	$P^I_S(t)$	.6695	9.140×10^{-5}	4.122×10^{-9}
2	$P^I_S(t)$	.6229	4.446×10^{-5}	1.037×10^{-9}
3	$P^I_S(t)$	.5791	2.034×10^{-5}	$.0205 \times 10^{-9}$

AKPPENDIX

In order to determine each pseudo-environment from $C_1, C_2, \ldots, C_m$, we need Lemma A.

Lemm A:
$C_1, C_2, \ldots, C_m$ may be re-identified as $D_1, D_2, \ldots, D_m$ so that: 1) $D_1, D_2, \ldots, D_k$ & $U_{i \ge K+1} D_i$ are pairwise disjoint and 2) $D_{K+1}, D_{K+2}, \ldots, D_m$ ar not pairwise disjoint for a unique integer K.

Proof: If all C_i's are pairwise non-disjoint, then $K = 0$, otherwise, choose a D_1 from them so that D_1 is disjoint from the rest of them. Now, if the rest of them are not pairwise disjoint, then $K = 1$, otherwise, choose a D_2 from them so that D_2 is disjoint from the rest of them. Continuing in athis way, we can finally reach the unique integer K. *Q.E.D.*

In order to calculate the weight of each pseudo-environment or to prove lemma E, we need to porve lemmas B,C,D.

Lemma B:

Let X & Y obey exponential distributions with intensities μ & ν respectively. Suppose that X & Y are s-independent. Then

$$r\{X \le t, X \le Y\} = \frac{\mu}{\mu+\nu}\,(1-\exp(-\mu+\nu)t).$$

Proof. $\Pr\{X \le t, X \le Y\} = \int_0^t \int_x^\infty \mu \cdot \nu \cdot \exp(-(\mu \cdot x + \nu \cdot y))dy \cdot dx = \mu/\mu+\nu\,(1-\exp(-\mu+\nu)t).$ *Q.E.D.*

Lemma C:

Let $X_1, X_2, \ldots$ & X_n be s-independent random variables and X be the minimum of such X_i follows exponential distribution with intensity μ_i for each $1 \le i \le n$. The X follows exponential distribution with intensity $\Sigma_i\, \mu_i$. □

Lemma D:

Let $C_1, C_2, \ldots, C_m$ be pairwise non-disjoint. The

$$\Pr\{\bar{C}_i \text{ occurs first during } [0,t]\} = \frac{\nu_i}{\Sigma_s\, \nu_s}\,[1-e^{-(\Sigma_s \nu_s)t}]$$

$$\Pr\{\text{no CCF during } [0,t]\} = e^{-(\Sigma_s \nu_s)t}.$$

Proof. The first equation follows by applying Lemmas B & C to the facts that $Y_i \equiv \min\{X_j : j \ne i\}$ is exponentially distributed with pdf $\Sigma_{s \ne i}\, \nu_s$ and $\Pr\{\bar{C}_i$ occurs first during $[0,t]\} = \Pr\{X_i \le t,\ X_i \le Y_i\}$. *Q.E.D.*

According to Lema D, we have:

Lemma E:

For each $j > K$,

$$\Pr\{\bar{C}_j\text{-first during } [0,t]\} = (\lambda_{C_j}/\Sigma_{i>K}\lambda_{C_i}) \cdot (1-\exp(-(\Sigma_{i>K}\lambda_{C_i})t))$$

$$\Pr\{\text{no } \bar{C}_i \text{ occurs during } [o,t] \text{ for all } i>K\} = \exp(-(\Sigma_{i>K}\lambda_{C_i})t))$$

ACKNOWLEDGMENT

This work is partially supported under NSC in Taiwan.

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Manuscript TR87-168 received 1987 September 14; revised 1988 September 7.

IEEE Log Number 27192

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A Decomposition Scheme for the Analysis of Fault Trees and Other Combinatorial Circuits

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Manuscript TR87-104 received 1987 July 28; revised 1988 November 22; revised 1989 January 2.

IEEE Log Number 27903

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